FM Synthesis. Technology, practice, legacy.

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1 History

1.1 Digital synthesis in early '80s

Early '80s is an era, when increase of computer's processing powers start to coincide with commercial available products on a broader scale. Till that time, many technologies of sound synthesis had been already developed theoretically or applied on stationary units.

Already in 1983, most of the common synthesis techniques used till today were already established, each with specific merits and drawbacks [7]. Some of those approaches (e.g.*fixed-waveform, subtractive* and *additive synthesis*) allowed to achieve similar goals that analogue systems aimed at, but doing away with space-taking physical equipment. Unfortunately, those techniques often traded off high quality and "living" output for the sake of comprehensive parameter list.

Other techniques like inclusion of random signals, *granular synthesis*, the Dutch *VOSIM*, digital *waveshaping* and digital *ring modulation* offered production of rich and complex timbres in exchange of the precision control, intuitive set of parameters or much needed stability in power and amplitude of the signal itself.

1.2 Yamaha DX7

Within this technological landscape, the FM synthesis adresses a lot of issues of its time. Being computationally relatively inexpensive, FM synthesis provides a volumewise stable signal characterised by timbral plasticity unheard of before. The set of control parameters, while still being difficult to use, does not require extensive technical knowledge (although the modulation itself as a phenomenon presents significant mathematical challenges).

In 1983, Yamaha introduced DX7 synthesizer based on Yamaha's own implementation of FM. Since its introduction to the music production market, FM sounds and DX7 presets became so ubiquitous that they constitute the very "sound of the'80s" [3].

2 FM synthesis

2.1 The "classic" FM synthesis

Despite the fact that mathematical mechanisms gouverning frequency modulation (FM) had been already well described before, it was John M. Chowning who succesfully applied those as a synthesis technique [2].

To explain the basics of it briefly, we need to introduce a general formula for an instantaneous amplitude of a sinewave signal (i.e. "volume" of a signal measured at a certain time):

$$s(t) = A \cdot sin(2\pi f t) \tag{1}$$

where: $s(t)$ = instantaneous amplitude,
A = amplitude scaling factor,
f = frequency in Hz,
t = a specific moment in time (in s).

To achieve the basic effect of FM synthesis, one must allow to control the frequency of a sinewave oscillator (*carrier*) by the signal of another one (*modulator*). Apart from the amplitude control of the output (A), it is possible to scale the influence of the modulator on carrier's output (its perceptual *timbre*) by scaling modulator's amplitude with a constant called *modulation index* (k):

$$s(t) = A \cdot cos(2\pi f_c t + k \cdot sin(2\pi f_m t)) \tag{2}$$

where: f_c = carrier frequency,
f_m = modulator frequency,
k = modulation index.

There are several merits of FM synthesis, when compared with other digital synthesis techniques at the beginning of '80s. First, it incorporates many advantages of the techniques mentioned by Poli, and achieves familiar signal transformations. For example, Lazzarini *et al.* demonstrated how to interpret a simple carrier-modulator output as a form of waveshaping [5].

Second, it maps intuitively many of the FM formula's components with the already well-recognizible synthesizer parameters. A and k may represent an instantaneous value of signal's envelope ("dynamics" unfolding in time); the former controls the "volume" while the latter changes the brightness of signal's "timbre".

Third, the $f_m : f_c$ encodes the information on the signal harmonicity. Deviation from simple integer ratio introduces inharmonic partials, allowing for a wide timbral variation.

2.2 Frequency understood as phase

It is noteworthy to point out, that within the aforementioned formulas, frequency is expressed by a rotating *angle* [7]. Thus, the modulator can correspond to the function $\phi(t)$, gouverning the angle increment at a given time:

$$s(t) = A \cdot cos(2\pi f_c t + \phi(t)) \tag{3}$$

Because the implementation of $\phi(t)$ might lie outside the acoustical understanding of *frequency*, the term *phase modulation* (PM) is sometimes adopted instead of FM.

It is also possible to use $\phi(t)$ as a lookup function querying a sampled wavetable (instead of calculating values on the spot). This technique — named *phase distortion synthesis* — was used in Casio CZ-1 synthesizer [5], a competitor to Yamaha DX7 (which implemented PM variant of FM).

2.3 Dilemma of Bessel function

Chowning noticed, that the spectral components, when in negative frequency domain, reflect around y-axis and alter the positive harmonics. This behaviour matches the FM synthesis description as a Bessel function — mathematical model of a vibrating membrane:

$$s(t) = A \cdot \sum_{n=-\infty}^{\infty} J_n(k)\ cos(2\pi f_c t + n \cdot 2\pi f_m t) \tag{4}$$

where: n = number of partial
$J_n(k)$ = Bessel function of the first kind.

Although Chowning described such spectra as "dynamic" and "living" [2], naturaly generated spectra does not behave this way [4]. This feature contributed to the so called timbral "digital coldness", an aestethical antithesis to the "analogue warm" characterising synths and Rhode pianos from '70s [3].

2.4 Modified FM

It is possible to modify the FM proceedure as described by Lazzarini & Timoney [4]. In the so called ModFM synthesis, the modulation index k is expressed by the negative imaginary number and the amplitude is normalized.

$$s(t) = \frac{A}{e^k} \cdot \sum_{n=-\infty}^{\infty} I_n(k)\ cos(2\pi f_c t + n \cdot 2\pi f_m t) \tag{5}$$

where: e^k = normalization of the amplitude scaling,
$I_n(k)$ = modified Bessel function (of nth order).

Lazzarini & Timoney reported timbre quality improvements when emulating evolving spectra of acoustic instruments (brass' attack phase, woodwind's modulations and — most spectacularly — bells). The resynthesized timbres required much higher distortion index to achieve similar levels of perceived brightness compared to Chowning's FM — one of the factors contributing to the feel of "brightness".

3 Taking control with Adaptive FM

When FM synthesis is aimed to be applied to the musical practice, be it studio work or live performance, many difficulties appear regarding the state of control of the signal and complex synthesis process.

We present three coinciding approaches toward formalizing control upon some of the more expressive qualities of the FM synthesis.

3.1 Adaptive FM

Lazzarini *et al.* explained, that an input signal itself might by manipulated as a carrier by a modulator after application of Hilbert transform on it within a model of phase distortion synthesis [5].

In a different study done by Lazzarini, a similar procedure was succesfully applied with the ModFM formulas [4].

3.2 Genetic Algorhythms

A different approach by using genetic algorhythms (GA) was applied by Macret and his research team, comparing the spectral input to the resynthesized output [6].

The team compared results of using FM and ModFM with parameters attuned by the GA. The ModFM turned out to be the quicker for the GA to calculate, providing better spectral tone approximations and rendering the output less bright. Macret also confirmed Lazzarini & Timoney's discovery [4] that adaptive algorhythms reach for lower values of modulation indices k.

3.3 Deep Neural Network

Last but not least situates the most recent work by Caspe *et al.* combining modern adaptive approach with the iconic legacy of FM synthesis — an architecture of the famous Yamaha DX7 synthesizer [1].

The group employed Deep Neural Network system to look for such parameters values which allow for the resynthesis of a given input signal. Such a method not only proved succesful, but also provided a new set of "sound design primitives" ("macro" parameters), allowing for complex but perceptually purposeful alteration of the synthesized timbre.

4 Conclusions

FM synthesis is a multi-paradigm phenomenon. To understand it means to build a link between mathematical equations, aural perception, aesthetical experience and cultural codes the importance of it emerges from.

The paper provides formulas for a simple FM synthesis. Their introduction is important when discussing various names of the procedure (pointing to different aspects and implementation subtleties).

The studies presented here introduce insights of getting closer to the founding "sound of the '80s" experience without modifying the expected premises, pursuing the original quest for the "living" timbre — achievable with new means (AG, DNN), but with "oldy goldy" Yamaha DX7 architecture and its derivatives.

References

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